hep-ph/9911370
IASSNS–HEP–99–106
WIS–99/36/Nov–DPP

Neutrino Parameters, Abelian Flavor Symmetries, and Charged Lepton Flavor Violation

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Neutrino masses and mixings have important implications for models of fermion masses, and, most directly, for the charged lepton sector. We consider supersymmetric Abelian flavor models, where neutrino mass parameters are related to those of charged leptons and sleptons. We show that processes such as $\tau \to \mu\gamma$, $\mu \to e\gamma$ and $\mu - e$ conversion provide interesting probes. In particular, some existing models are excluded by current bounds, while many others predict rates within reach of proposed near future experiments. We also construct models in which the predicted rates for charged lepton flavor violation are below even the proposed experimental sensitivities, but argue that such models necessarily involve loss of predictive power.

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1. Introduction

The most problematic aspect of the Standard Model is the unnaturally small ratio between the electroweak breaking scale and the Planck scale, that is, the fine-tuning problem. Supersymmetry protects this ratio against radiative corrections. Another puzzling aspect of the Standard Model is the unexplained hierarchy in the fermion masses and mixings, that is, the flavor puzzle. Approximate horizontal symmetries give rise to selection rules that can account for this hierarchy. The framework of *supersymmetric flavor models*, which combines these two extensions of the Standard Model, is particularly interesting because there is an interplay between the two ingredients. Supersymmetry affects the flavor parameters since it requires the Yukawa parameters to be holomorphic. Horizontal symmetries affect the supersymmetry breaking parameters since these parameters are subject to appropriate selection rules. One of the most attractive features of supersymmetric flavor models is that, as a result of this interplay, the measured values of fermion masses and mixings have implications for supersymmetric contributions to flavor changing neutral current processes. Conversely, measurements of rare processes provide possibly stringent tests of models with horizontal symmetries.

Recent measurements of the fluxes of atmospheric [1] and solar [2] neutrinos have added to our knowledge of neutrino parameters. The simplest interpretation of the experimental results concerning atmospheric neutrinos (AN) is in terms of $\nu_\mu - \nu_\tau$ oscillations, with the following central values for the mass-squared difference and mixing angle:

$$\Delta m^2_{23} \sim 2 \times 10^{-3}\ \mathrm{eV}^2, \quad \sin^2 2\theta_{23} \sim 1. \tag{1.1}$$

The simplest interpretation of the experimental results concerning solar neutrinos (SN) is in terms of $\nu_e - \nu_x$ ($x = \mu$ or τ) oscillations, with one of the following sets of parameters:

$$\begin{array}{ccc} & \Delta m^2_{1x}\ [\mathrm{eV}^2] & \sin^2 2\theta_{1x} \\ \\ \mathrm{MSW(SA)} & 5\times 10^{-6} & 0.006 \\ \mathrm{MSW(LA)} & 2\times 10^{-5} & 0.8 \\ \mathrm{VO} & 8\times 10^{-11} & 0.8 \end{array} \tag{1.2}$$

Here MSW (VO) refers to matter-enhanced (vacuum) oscillations, and SA (LA) stands for a small (large) mixing angle. These neutrino parameters, together with the masses of the

charged leptons,

$$m_e \simeq 0.51 \text{ MeV}, \quad m_\mu \simeq 106 \text{ MeV}, \quad m_\tau \simeq 1777 \text{ MeV}, \tag{1.3}$$

constrain model building with horizontal symmetries in the lepton sector.

While any measurement of neutrino parameters provides welcome guidance for attempts to explain the fermion masses, the parameters of Eqs. (1.1) and (1.2) are particularly provocative. In the simplest realizations of models with horizontal symmetries, states with large mixing must have similar masses. In contrast, (1.1) and (1.2) suggest both a large 2-3 mixing and a hierarchically suppressed m_2/m_3 ratio in the neutrino sector. (Here we assume that the mass-squared differences are of order of the larger mass-squared involved.) The experimental data has thus motivated many studies of extensions of, or alternatives to, the simplest models. The possibilities explored include:

(i) The neutrino masses arise from different sources. For example, in the framework of supersymmetry without R-parity, the heaviest neutrino acquires its mass at tree level while the lighter ones become massive only through loop effects.

(ii) Certain Yukawa couplings vanish because of holomorphy ('holomorphic zeros').

(iii) The horizontal symmetry is discrete.

(iv) The horizontal symmetry is broken by two small breaking parameters of equal magnitude and opposite charge.

(v) The hierarchy in masses is accidental.

While there are clearly many possibilities, most of these models solve the 'large mixing–large hierarchy' problem in the following way: the neutrino mass hierarchy follows from the structure of the neutrino mass matrix, while the large mixing arises from diagonalizing the charged lepton mass matrix. With the standard model fermion content alone, this approach produces the desired neutrino properties with no other experimentally interesting implications. For example, although the mixing of neutrinos induces, at the loop-level, flavor mixing in the charged leptons, it is at an unobservably small level: for neutrino masses of 100 eV, $B(\mu \to e\gamma) < 10^{-40}$ [3].

When extended to *supersymmetric* models, however, the large mixing in the charged lepton mass matrix has profound experimental implications. Supersymmetry introduces

additional scalars that are governed by the same horizontal symmetries. As we shall see, these scalars can induce charged lepton flavor violation (LFV) at current experimental sensitivities. For example, the large mixing angle suggested by AN implies a rate for $\tau \to \mu\gamma$ that is typically close to the present bound if slepton masses are around 100 GeV. Thus, the new results on neutrino parameters warrant a close analysis of charged LFV in the context of supersymmetric flavor models.

At present, searches for charged LFV have yielded only upper bounds. Among the most stringent are the bounds on radiative decay [4,5,6],

$$\begin{aligned} B(\mu \to e\gamma) &\leq 1.2 \times 10^{-11}, \\ B(\tau \to e\gamma) &\leq 2.7 \times 10^{-6}, \\ B(\tau \to \mu\gamma) &\leq 1.1 \times 10^{-6}, \end{aligned} \tag{1.4}$$

and the bound on $\mu - e$ conversion [7],

$$\frac{\sigma(\mu^- \mathrm{Ti} \to e^- \mathrm{Ti})}{\sigma(\mu^- \mathrm{Ti} \to \mathrm{capture})} < 6.1 \times 10^{-13}. \tag{1.5}$$

In the future, all of these sensitivities are likely to improve. Particularly promising are those involving muon decay and conversion [8]: for example, a future experiment at PSI will be sensitive to $B(\mu \to e\gamma)$ at the 10^{-14} level [9], and the MECO collaboration has proposed an experiment to probe $\mu - e$ conversion down to 5×10^{-17}, four orders of magnitude beyond present sensitivities [10]. In models where LFV is mediated dominantly by a photon, as in the supersymmetric models discussed here, these rates are related by [11]

$$\frac{\sigma(\mu^- \mathrm{Ti} \to e^- \mathrm{Ti})}{\sigma(\mu^- \mathrm{Ti} \to \mathrm{capture})} \approx 0.003\ B(\mu \to e\gamma). \tag{1.6}$$

In the following, for brevity, we present predictions for decay rates only, but it should be understood that the current bounds from and prospects for $\mu \to e\gamma$ and $\mu - e$ conversion are competitive.

The purpose of this work is to understand the implications of the lepton flavor parameters of Eqs. (1.1), (1.2) and (1.3) for the lepton flavor changing processes of Eqs. (1.4) and (1.5). We focus on the framework of supersymmetric Abelian horizontal symmetries. (Similar issues have been investigated within different supersymmetric flavor models in

Refs. [12-14].) Our work is closely related to that of Refs. [15,16] where the various classes of models of Abelian flavor symmetries that can accommodate (1.1) and (1.2) were presented. Here we analyze the consequences of these classes of models for charged LFV. We will try to answer the following three questions:

(i) Are any of the relevant flavor models excluded by the upper bounds on lepton flavor changing processes?

(ii) Are there generic (or, preferably, model-independent) predictions for such processes in this framework that can be tested in the future?

(iii) Is it possible to construct Abelian flavor models that predict charged LFV far below even future sensitivities, and if so, how complicated are these models?

Before we describe the details of our study, we emphasize that our basic, underlying assumption is that Abelian flavor symmetries determine the structure of both Yukawa couplings and supersymmetry breaking parameters. (When we discuss models in which there are additional ingredients that affect the hierarchy in the flavor parameters, such as models without R-parity, we will state so explicitly.) It could be, however, that Abelian flavor symmetries determine the structure of the Yukawa couplings, but their effects on the supersymmetry breaking parameters are screened. This is the case, for example, if the slepton masses are dominated by large, universal contributions from renormalization group evolution. For squarks, the universal contribution from gaugino masses could easily be dominant. For sleptons, however, the effects are much weaker since here the renormalization is driven by α_2 instead of α_3 [17,18]. A more likely case that would lead to slepton mass universality is supersymmetry breaking that is mediated at some low energy in a flavor-blind way, as in gauge-mediated supersymmetry breaking. In such cases, our study does not apply. In particular, when we state that various models in the literature are excluded, we base our statements on the above assumption. Most of these models are still viable models of neutrino parameters if slepton masses are approximately universal.

2. General Considerations

2.1. Supersymmetric contributions to charged LFV

Supersymmetric models provide, in general, new sources of flavor violation. These are most commonly analyzed in the basis in which the charged lepton mass matrix and the gaugino vertices are diagonal. In this basis, the slepton masses are not necessarily flavor-diagonal and have the form

$$\tilde{\ell}^*_{M\,i}(M^2_{\tilde{\ell}})^{MN}_{ij}\tilde{\ell}_{N\,j} = (\tilde{\ell}^*_{L\,i} \quad \tilde{\ell}^*_{R\,k}) \begin{pmatrix} M^2_{L\,ij} & A_{il}v_d \\ A_{jk}v_d & M^2_{R\,kl} \end{pmatrix} \begin{pmatrix} \tilde{\ell}_{L\,j} \\ \tilde{\ell}_{R\,l} \end{pmatrix} , \tag{2.1}$$

where $M, N = L, R$ label chirality, and $i, j, k, l = 1, 2, 3$ are generational indices. M^2_L and M^2_R are the supersymmetry breaking slepton masses. The A parameters enter in the trilinear scalar couplings $A_{ij}\phi_d\tilde{\ell}_{Li}\tilde{\ell}^*_{Rj}$, where ϕ_d is the down-type Higgs boson, and $v_d = \langle\phi_d\rangle$. We neglect small flavor-conserving terms involving $\tan\beta$, the ratio of Higgs vacuum expectation values.

In this basis, charged LFV takes place through one or more slepton mass insertion. Each mass insertion brings with it a factor of $\delta^{MN}_{ij} \equiv (M^2_{\tilde{\ell}})^{MN}_{ij}/\tilde{m}^2$, where $\tilde{m}^2$ is the representative slepton mass scale. Physical processes therefore constrain

$$(\delta^{MN}_{ij})_{\text{eff}} \sim \max\left[\delta^{MN}_{ij},\ \delta^{MP}_{ik}\delta^{PN}_{kj},\ \ldots,\ (i \leftrightarrow j)\right] . \tag{2.2}$$

For example,

$$(\delta^{LR}_{12})_{\text{eff}} \sim \max\left[A_{12}v_d/\tilde{m}^2,\ M^2_{L1k}A_{k2}v_d/\tilde{m}^4,\ A_{1k}v_dM^2_{Rk2}/\tilde{m}^4,\ \ldots,\ (1 \leftrightarrow 2)\right] . \tag{2.3}$$

Note that contributions with two or more insertions may be less suppressed than those with only one.

In models with horizontal symmetries, Yukawa and supersymmetry breaking parameters are ambiguous up to $\mathcal{O}(1)$ factors. It is therefore sufficient to obtain order of magnitude estimates for the supersymmetric contributions to radiative lepton decay. These have been

analyzed in [19]. Normalizing these results to the current bounds, we find

$$\begin{aligned}
\frac{B(\mu \to e\gamma)}{1.2 \times 10^{-11}} &\sim \max\left[\left(\frac{(\delta^{LL}_{12})_{\rm eff}}{2.0 \times 10^{-3}}\right)^2, \left(\frac{(\delta^{LR}_{12})_{\rm eff}}{6.9 \times 10^{-7}}\right)^2\right]\left(\frac{100\ \text{GeV}}{\tilde{m}}\right)^4, \\
\frac{B(\tau \to e\gamma)}{2.7 \times 10^{-6}} &\sim \max\left[\left(\frac{(\delta^{LL}_{13})_{\rm eff}}{2.2}\right)^2, \left(\frac{(\delta^{LR}_{13})_{\rm eff}}{1.3 \times 10^{-2}}\right)^2\right]\left(\frac{100\ \text{GeV}}{\tilde{m}}\right)^4, \\
\frac{B(\tau \to \mu\gamma)}{1.1 \times 10^{-6}} &\sim \max\left[\left(\frac{(\delta^{LL}_{23})_{\rm eff}}{1.4}\right)^2, \left(\frac{(\delta^{LR}_{23})_{\rm eff}}{8.3 \times 10^{-3}}\right)^2\right]\left(\frac{100\ \text{GeV}}{\tilde{m}}\right)^4.
\end{aligned} \tag{2.4}$$

Here, the lightest neutralino is assumed to be photino-like, and so bounds on $(\delta^{LL}_{ij})_{\rm eff}$ apply also to $(\delta^{RR}_{ij})_{\rm eff}$. We have set $m^2_{\tilde{\gamma}}/\tilde{m}^2 = 0.3$. The bounds are fairly insensitive to this ratio; for example, even for $m^2_{\tilde{\gamma}}/\tilde{m}^2 = 1$, the bounds on δ^{LL} and δ^{LR} are only weakened by factors of ~ 2 and 1.2, respectively [19].

Finally, we note that in the physical basis with diagonal charged lepton and slepton masses, flavor violation appears in the gaugino vertices $K^{MN}_{ij}\tilde{\gamma}\ell_{M\,i}\tilde{\ell}^*_{N\,j}$. (Of course, left- and right-handed sleptons mix; in our notation here, the slepton mass eigenstates are labeled by their dominant chirality.) The mass insertion parameters are related to these mixing angles by $(\delta^{MN}_{ij})_{\rm eff} \sim \max\left[K^{MP}_{ik}K^{NP}_{jk},\ (i \leftrightarrow j)\right]$.

2.2. Models of Abelian flavor symmetries

We are interested in finding the relevant $(\delta^{MN}_{ij})_{\rm eff}$ parameters in the framework of approximate Abelian flavor symmetries. We have in mind theories with a spontaneously broken horizontal symmetry of one of the following three types: (*i*) An anomalous $U(1)$ symmetry where the anomaly is cancelled by the Green-Schwarz mechanism; (*ii*) A discrete Z_n symmetry; (*iii*) A non-anomalous $U(1)$. The details of these full high energy theories are not important: for our purposes, it is sufficient to consider low energy effective theories with a horizontal symmetry that is explicitly broken by a small parameter. The three types of such models that we consider are, however, motivated by the high energy theories described above. We define the models by the selection rules that apply to the low energy effective theory:

(*i*) *A $U(1)$ symmetry broken by a single breaking parameter.* We denote the breaking parameter by λ and assign to it a horizontal charge -1. Then the following selection rules apply:

a. Terms in the superpotential that carry an integer $U(1)$ charge $n \geq 0$ are suppressed by λ^n. Terms with $n < 0$ vanish by holomorphy.

b. Terms in the Kähler potential that carry an integer $U(1)$ charge n are suppressed by $\lambda^{|n|}$.

(ii) A Z_m symmetry broken by a single breaking parameter. We denote the breaking parameter by λ and assign to it a horizontal charge -1. Then the following selection rules apply:

a. Terms in the superpotential that carry an integer Z_m charge n are suppressed by $\lambda^{n \pmod{m}}$.

b. Terms in the Kähler potential that carry an integer Z_m charge n are suppressed by $\lambda^{\min[|n|, n \pmod{m}]}$.

(iii) A $U(1)$ symmetry broken by two breaking parameters. We denote the breaking parameters by λ and $\bar{\lambda}$. They have equal magnitude, $\lambda = \bar{\lambda}$, and carry opposite horizontal charge, $+1$ and -1, respectively. Then the following selection rules apply:

a. Terms in the superpotential and in the Kähler potential that carry an integer $U(1)$ charge n are suppressed by $\lambda^{|n|}$.

In all cases, terms in both the superpotential and Kähler potential with non-integer horizontal charge vanish.

To be specific, we set $\lambda \sim 0.2$ and require that our models are consistent with (1.1), (1.2) and (1.3), namely that they give the following parametric suppression for the lepton flavor parameters:

$$V^\ell_{23} \sim 1, \quad V^\ell_{13} \lesssim \lambda, \quad V^\ell_{12} \sim \begin{cases} 1 & \text{MSW(LA), VO} \\ \lambda^2 & \text{MSW(SA)} \end{cases}, \tag{2.5}$$

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \sim \begin{cases} \lambda^2 - \lambda^4 & \text{MSW} \\ \lambda^8 - \lambda^{12} & \text{VO} \end{cases}, \tag{2.6}$$

$$m_\tau / \langle \phi_d \rangle \sim \lambda^3 - 1, \quad m_\mu / m_\tau \sim \lambda^2, \quad m_e / m_\mu \sim \lambda^3. \tag{2.7}$$

In (2.6), we allow a large range for the VO option, since when observables depend on a very high power of λ, the sensitivity to the precise value of the breaking parameter is enhanced. In (2.7), the range for the Yukawa coupling of the tau corresponds to values of $\tan\beta$ between 1 and m_t/m_b.

2.3. A naive estimate

Before we analyze specific classes of models of Abelian flavor symmetries, let us introduce a naive estimate of the $(\delta^{MN}_{ij})_{\rm eff}$ parameters in this framework. By 'naive estimate' we mean that we make an order of magnitude estimate in models with the following features:

a. The horizontal symmetry is a single $U(1)$;

b. The symmetry is broken by a single parameter;

c. Holomorphic zeros play no role;

d. Singlet neutrinos play no role.

We emphasize that such 'naive models' cannot accommodate the neutrino parameters of Eqs. (1.1) and (1.2). In particular, as alluded to previously and as will become clear, such models cannot explain large mixings between states with hierarchically different masses. Therefore, we should not expect that our naive predictions necessarily hold in all viable models. However, naive models and viable models often share several important features, and so it is a worthwhile exercise to consider first the simpler case of naive models.

In this framework, we have the following order of magnitude estimates in the interaction basis, where the horizontal charges are well-defined:

$$\begin{aligned}
(M_\ell)_{ij} &\sim v_d \lambda^{H(L_i)+H(\bar\ell_j)+H(\phi_d)}, \\
(M_\nu)_{ij} &\sim \frac{v_u^2}{M} \lambda^{H(L_i)+H(L_j)+2H(\phi_u)}, \\
(M^2_L)_{ij} &\sim \tilde m^2 \lambda^{|H(L_i)-H(L_j)|}, \\
(M^2_R)_{ij} &\sim \tilde m^2 \lambda^{|H(\bar\ell_i)-H(\bar\ell_j)|}, \\
A_{ij} &\sim \tilde m \lambda^{H(L_i)+H(\bar\ell_j)+H(\phi_d)}.
\end{aligned} \tag{2.8}$$

Here L_i are lepton doublets and $\bar\ell_i$ are charged lepton singlets. The Higgs vacuum expectation values are denoted by v_u and v_d, and M is some large mass scale. Hypercharge and Peccei-Quinn U(1) symmetries may be used to set the charges of both Higgs bosons to zero, and we indeed do so in all the explicit models described below.[2]

[2] Note, however, that the Peccei-Quinn symmetry is not a symmetry of the full theory. It is broken explicitly by the μ term. It is only an accidental symmetry of the Yukawa sector, of the A-couplings and of the slepton mass-squared terms. Shifts by the Peccei-Quinn charge do not affect these sectors and we can use them for our purposes.

From (2.8), one can deduce the order of magnitude of the physical parameters. In particular, for the mixing angles in the $W^\pm$ couplings to neutrinos and charged leptons V^ℓ_{ij} (that is, the MNS matrix [20]) and for the mixing angles in the photino $\tilde\gamma$ couplings to charged leptons and charged sleptons, K^{MN}_{ij}, we have

$$\begin{aligned} V^\ell_{ij} &\sim \lambda^{|H(L_i)-H(L_j)|}, \\ K^{LL}_{ij} &\sim \lambda^{|H(L_i)-H(L_j)|}, \\ K^{RR}_{ij} &\sim \lambda^{|H(\bar\ell_i)-H(\bar\ell_j)|}, \\ K^{LR}_{ij} &\sim (v_d/\tilde m)\lambda^{H(L_i)+H(\bar\ell_j)+H(\phi_d)}, \end{aligned} \tag{2.9}$$

while for the fermion masses we have

$$\begin{aligned} m(\ell^\pm_i) &\sim v_d\lambda^{H(L_i)+H(\bar\ell_i)+H(\phi_d)}, \\ m(\nu_i) &\sim \frac{v_u^2}{M}\lambda^{2H(L_i)+2H(\phi_u)}. \end{aligned} \tag{2.10}$$

Equations (2.9) and (2.10) demonstrate in a clear way how the fermion flavor parameters (masses and mixing) are related to the supersymmetric flavor violation. Explicitly, we get:

$$\begin{aligned} K^{LL}_{ij} &\sim V^\ell_{ij}, \\ K^{RR}_{ij} &\sim \frac{m(\ell^\pm_i)}{m(\ell^\pm_j)V^\ell_{ij}}, \\ K^{LR}_{ij} &\sim \frac{m(\ell^\pm_j)V^\ell_{ij}}{\tilde m}, \text{ where } H(L_i) > H(L_j) \text{ and } H(\bar\ell_i) > H(\bar\ell_j) \\ K^{LR}_{ji} &\sim \frac{m(\ell^\pm_i)}{V^\ell_{ij}\tilde m}, \quad \text{where } H(L_i) > H(L_j) \text{ and } H(\bar\ell_i) > H(\bar\ell_j). \end{aligned} \tag{2.11}$$

We can use Eqs. (1.1), (1.2) and (1.3) to make naive predictions for the relevant supersymmetric mixing angles. For the $\tau\to\mu$ transitions, we have:

$$\begin{aligned} (\delta^{LL}_{23})_{\rm eff} &\sim V^\ell_{23} \sim 1, \\ (\delta^{LR}_{23})_{\rm eff} &\sim \frac{m_\tau V^\ell_{23}}{\tilde m} \sim 0.02\left(\frac{100\ \text{GeV}}{\tilde m}\right). \end{aligned} \tag{2.12}$$

For the $\mu\to e$ transitions, we have for both MSW(LA) and VO:

$$\begin{aligned} (\delta^{LL}_{12})_{\rm eff} &\sim V^\ell_{12} \sim 1, \\ (\delta^{LR}_{12})_{\rm eff} &\sim \frac{m_\mu V^\ell_{12}}{\tilde m} \sim 10^{-3}\left(\frac{100\ \text{GeV}}{\tilde m}\right), \end{aligned} \tag{2.13}$$

and for MSW(SA):

$$
\begin{aligned}
(\delta^{LL}_{12})_{\rm eff} &\sim V^\ell_{12} \sim 0.04, \\
(\delta^{LR}_{12})_{\rm eff} &\sim \frac{m_e}{V^\ell_{12}\tilde m} \sim 10^{-4}\left(\frac{100\ \text{GeV}}{\tilde m}\right).
\end{aligned}
\tag{2.14}
$$

For the $\tau \to e$ transitions, we have for both MSW(LA) and VO:

$$
\begin{aligned}
(\delta^{LL}_{13})_{\rm eff} &\sim V^\ell_{12}V^\ell_{23} \sim 1, \\
(\delta^{LR}_{13})_{\rm eff} &\sim \frac{m_\tau V^\ell_{13}}{\tilde m} \sim 0.02\left(\frac{100\ \text{GeV}}{\tilde m}\right),
\end{aligned}
\tag{2.15}
$$

and for MSW(SA):

$$
\begin{aligned}
(\delta^{LL}_{13})_{\rm eff} &\sim V^\ell_{12}V^\ell_{23} \sim 0.04, \\
(\delta^{LR}_{13})_{\rm eff} &\sim \frac{m_\tau V^\ell_{13}}{\tilde m} \sim 10^{-3}\left(\frac{100\ \text{GeV}}{\tilde m}\right).
\end{aligned}
\tag{2.16}
$$

We emphasize that the parameters of order one could be accidentally large or small, leading to an incorrect 'translation' of the experimental numbers to powers of λ in Eqs. (2.5)$-$(2.7), or to deviations of the $(\delta^{MN}_{ij})_{\rm eff}$ from the numerical estimates in Eqs. (2.12)$-$(2.16). These ambiguities constitute a limitation to the predictive power of this framework. We avoid, however, part of this ambiguity by presenting our estimates of the $(\delta^{MN}_{ij})_{\rm eff}$ parameters in Eqs. (2.12)$-$(2.16) in terms of mixing angles and mass ratios rather than directly in powers of λ. Our point is that the parametric suppression of the slepton flavor parameters is the same as that of the corresponding combinations of lepton flavor parameters. This statement is independent of the parameters of order one.

A comparison of the naive estimates of Eqs. (2.12)–(2.16) with Eq. (2.4) leads to the following conclusions:

(i) The MSW(LA) and VO solutions of the solar neutrino problem cannot be accommodated.

(ii) The MSW(SA) can be accommodated if the sleptons are heavier than $\mathcal{O}(500\ \text{GeV})$. The rate of the $\mu \to e\gamma$ decay should be close to the present bound.

(iii) The rate of the $\tau \to \mu\gamma$ decay should be not far below the present bound. (It is within one order of magnitude of the present bound if $\tilde m \lesssim 200\ GeV$, but falls like $1/\tilde m^6$ up to values of $\tilde m \sim 350\ GeV$ and like $1/\tilde m^4$ for higher values.)

As noted above, 'naive models,' which obey the conditions a-d specified above, cannot accommodate the neutrino parameters of Eqs. (1.1) and (1.2). In particular, the first

relation of Eq. (2.9) and the last of Eq. (2.10) require highly mixed neutrinos to have similar masses. Note, however, that Eqs. (2.12)−(2.16) relate the $(\delta_{ij}^{MN})_{\rm eff}$ mixing angles to the MNS mixing angles V_{ij}^{ℓ} and charged lepton masses, but not to the neutrino masses. Many of the viable extensions of the naive models modify Eq. (2.10) for the neutrino mass ratios but not Eq. (2.9) for the mixing angles. Consequently, the naive estimates remain valid in a large class of models. For such models, our analysis in this section gives the following important lessons:

1. Models of MSW(LA) or of VO where the naive predictions for $(\delta_{12}^{LL})_{\rm eff}$ and $(\delta_{12}^{LR})_{\rm eff}$ hold are excluded.
2. Both $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays provide interesting probes of Abelian flavor symmetries. The $\tau \to e\gamma$ decay is, at present, less sensitive to this type of new physics.

2.4. Pseudo-Dirac neutrinos

The AN data imply a large, maybe maximal mixing in the 23 subspace. The MSW(LA) and VO solutions of the SN problem require large mixing in the 12 space. It is an interesting possibility then that two of the neutrinos form a pseudo-Dirac neutrino, which would yield close to maximal mixing. This scenario becomes even more attractive in the framework of Abelian horizontal symmetries, because the symmetry could easily force two neutrinos into a pseudo-Dirac structure [21]. Take, for example, a horizontal $U(1)$ symmetry where two lepton doublets carry opposite charges (and $H(\phi_u) = 0$). Then, the corresponding off-diagonal terms in the Majorana mass matrix carry no $U(1)$ charge and are therefore unsuppressed by the horizontal symmetry. The diagonal terms, on the other hand, carry horizontal charges, and are either suppressed or forbidden. To be specific, take $H(L_2) = -1$ and $H(L_1) = +1$. Then, in the 12 subspace,

$$M_\nu \sim \frac{v_u^2}{M}\begin{pmatrix} \lambda^2 & 1 \\ 1 & 0 \end{pmatrix}, \tag{2.17}$$

yielding $\sin^2 2\theta_{12}^\nu \simeq 1$.

In Ref. [16] it was argued, however, that in Abelian flavor models that satisfy both (1.1) and (1.2), the pseudo-Dirac structure cannot apply to the 23 subspace. It can only

be relevant then to the 12 subspace, corresponding to either the MSW(LA) or the VO solution of the SN problem.

The interesting point about the case where ν_e and ν_μ form a pseudo-Dirac neutrino is that the large mixing, $V_{12}^\ell \sim 1$, comes from the neutrino sector. It is therefore not necessary that the charged lepton sector induce large 12 mixing. In fact, $\sin\theta_{12}^\ell \ll 1$ is unavoidable in models where a horizontal $U(1)$ is broken by a single parameter and it is generic (though not unavoidable) in models where the symmetry is broken by two small parameters of opposite signs. The naive predictions of (2.13) are therefore avoided.

We learn that if, in the future, measurements of SN make a convincing case for a (close to) maximal mixing but $\mu \to e\gamma$ is not observed, then a pseudo-Dirac structure for the corresponding neutrinos induced by an Abelian flavor symmetry can provide a very attractive explanation for this situation.

This statement is particularly relevant to the case of VO. In the case of MSW(LA), a truly maximal mixing is disfavored (see, *e.g.*, the discussion in Ref. [22]). If ν_e and ν_μ form a pseudo-Dirac neutrino, a sufficient deviation from maximal mixing can be induced by $\sin\theta_{12}^\ell$:

$$\sin\theta_{12}^\nu = \sqrt{2}/2, \quad \sin\theta_{12}^\ell \gtrsim 0.3 \quad \Longrightarrow \quad \sin^2 2\theta_{12} = 1 - \mathcal{O}(0.1). \tag{2.18}$$

However, in such a scenario, we have

$$\begin{aligned} (\delta_{12}^{LL})_{\rm eff} &\sim 0.3, \\ (\delta_{12}^{LR})_{\rm eff} &\sim 3\times 10^{-4}\left(\frac{100\ {\rm GeV}}{\tilde m}\right). \end{aligned} \tag{2.19}$$

Comparing (2.19) and (2.4), we conclude that viable MSW(LA) models with pseudo-Dirac structure in the 12 subspace require $\tilde m \gtrsim 1$ TeV and are, therefore, disfavored.

3. Specific Models

The naive models discussed above cannot accommodate the neutrino parameters of (1.1) and (1.2). In this section, we survey specific supersymmetric models with Abelian flavor symmetries that have been constructed to accommodate these parameters, and find their predictions for the lepton flavor violating decays (1.4).

3.1. *Accidental mass hierarchy*

As mentioned above, the main problem in accommodating the neutrino parameters is to have simultaneously a large mixing, $V_{23}^{\ell} \sim 1$, and a large hierarchy, $m_2/m_3 \lesssim 0.1$. In the case of MSW solutions to the solar neutrino problem, however, the hierarchy is close to 0.1 and could be simply accidental. By 'accidental' we mean that the hierarchy does not result from suppression by a small symmetry breaking parameter. Instead, it is the result of an accidental cancellation between $\mathcal{O}(1)$ coefficients, *e.g.*, $ac - b^2 = \mathcal{O}(0.1)$ with $a, b, c = \mathcal{O}(1)$. Such models generically allow a situation where L_2 and L_3 carry the same horizontal charge. In this case, we have

$$(M_\ell)_{23}/(M_\ell)_{33} \sim 1 \quad \Longrightarrow \quad (\delta_{23}^{LL})_{\rm eff} \sim 1, \quad (\delta_{23}^{LR})_{\rm eff} \sim 0.02 \left(\frac{100 \text{ GeV}}{\tilde{m}}\right), \tag{3.1}$$

as in the naive prediction of Eq. (2.12). Therefore, a generic (though not an unavoidable) prediction of this class of models is that, if charged slepton masses are not much higher than 100 GeV, $B(\tau \to \mu\gamma)$ should be close to the upper bound. As concerns $B(\mu \to e\gamma)$, there is no generic prediction here.

Abelian flavor models of this type were constructed in Refs. [23,24]. Equation (3.1) holds, indeed, in these models. The explicit models are constructed to accommodate the MSW(SA) solution of the SN problem. They actually give a rather small value for the relevant mixing angle, that is,

$$(M_\ell)_{12}/(M_\ell)_{22} \sim \lambda^3 \quad \Longrightarrow \quad (\delta_{12}^{LL})_{\rm eff} \sim 8 \times 10^{-3}, \quad (\delta_{12}^{LR})_{\rm eff} \sim 6 \times 10^{-4} \left(\frac{100 \text{ GeV}}{\tilde{m}}\right). \tag{3.2}$$

(The naive estimates (2.14) hold in these models, except that the value of V_{12} is smaller than what is implied by (1.2). The appropriate mixing in the charged current interactions is accidentally enhanced by about one order of magnitude.) These models are then viable, provided that slepton masses are rather heavy, $\tilde{m} \gtrsim 500$ GeV.

3.2. *Neutrino masses from different sources*

Different neutrino masses could come from different sources, so that the mass hierarchy is determined not only by the horizontal symmetry. In such a framework, neutrinos

with the same horizontal charge, and therefore with large mixing, may nevertheless have their masses hierarchically separated. The problem of accommodating (1.1) and (1.2) is then solved. It is then generic (though, again, not unavoidable) in these models that the horizontal charges of L_2 and L_3 are equal, leading to (3.1) and, consequently, to $B(\tau \to \mu\gamma)$ close to the bound.

A framework where this is the case is that of supersymmetry without R-parity. The Abelian horizontal symmetry could replace R-parity in suppressing dangerous lepton-number violating couplings [25]. If the B- and μ-terms are not aligned, one neutrino acquires its mass at tree level by mixing with neutralinos, while the other two acquire masses at the loop level. Explicit flavor models of this type were constructed, for example, in Refs. [26,27,28]. Equation (3.1) holds in these models. The explicit models are constructed to accommodate the MSW(SA) solution of the SN problem and Eq. (2.14) holds. (Ref. [27] assumes that supersymmetry breaking is gauge-mediated, in which case slepton masses are degenerate and the radiative lepton decays are highly suppressed.)

3.3. See-saw enhancement

A neutrino mass could be enhanced beyond the naive expectation by the see-saw mechanism. Singlet neutrinos with masses below the scale of horizontal symmetry breaking could induce such an enhancement [15]. In such a framework it is, again, possible that L_2 and L_3 carry the same horizontal charges. Consequently, the generic prediction is that of Eq. (3.1).

The idea of see-saw enhancement was presented in Ref. [22]. The horizontal symmetry is simply $L_e - L_\mu - L_\tau$. Under this symmetry, $H(L_2) = H(L_3) = -1$ and, consequently, (2.12) holds.

Interestingly, the symmetry forces a pseudo-Dirac neutrino in the 12 subspace. The large V^ℓ_{12} comes from the neutrino sector. The 12 mixing in the charged lepton sector is parametrically suppressed but, when fitted to MSW(LA) parameters, it is found that numerically the suppression is mild. Equation (2.19) holds, so that the model is viable only for very high slepton masses, $\tilde{m} \gtrsim 1$ TeV.

Another model in Ref. [22], within the class of see-saw enhancement, is constructed

to fit the MSW(SA) parameters. The naive predictions (2.12) and (2.14) for, respectively, $(\delta_{23}^{MN})_{\rm eff}$ and $(\delta_{12}^{MN})_{\rm eff}$, are valid. This model requires then that $\tilde{m} \gtrsim 500$ GeV.

A framework where a horizontal $U(1)$ is combined with an $SU(5)$ grand unified theory is presented in Refs. [29,30]. Here, $H(L_2) = H(L_3)$ and the hierarchy of masses is induced by see-saw enhancement. A large 23 mixing, as in (2.12), is predicted. As concerns the 12 mixing, the largest effect comes from $\delta_{12}^{RR} \sim \lambda$. These specific models are then also only viable for very high slepton masses, $\tilde{m} \gtrsim 1$ TeV.

A framework where a horizontal $U(1)$ is broken into an exact lepton parity was presented in Ref. [31]. For the neutrino spectrum, the mechanism of see-saw enhancement is in operation. In the specific model presented in [31], $H(L_2) = H(L_3)$ and (2.12) holds. While the model is constructed to fit the MSW(SA) parameters, it has $H(\bar{\ell}_1) = H(\bar{\ell}_2)$, leading to a surprisingly large 12 mixing, $\delta_{12}^{RR} \sim 1$. This specific model is then excluded.

3.4. Holomorphic zeros

Holomorphy could induce a strong suppression of a neutrino mass ratio, compared to the naive estimate. This mechanism was proposed and viable models were constructed in Ref. [15]. In the models of Ref. [15], singlet neutrinos play no role. Then, the mass matrix for the active neutrinos in the 23 subspace is near-diagonal, and the large 23 mixing must arise from the charged lepton sector. Eq. (2.12) then holds independently of the details of the model, and $\tau \to \mu\gamma$ may be near its current bound.

As concerns the SN problem, the structure of the neutrino mass matrix allows only a pseudo-Dirac structure in the 12 subspace. The predictions of (2.19) hold in the MSW(LA) case, requiring $\tilde{m} \gtrsim 1$ TeV for the model to be viable. In the VO case, $(\delta_{12}^{MN})_{\rm eff}$ could be very small and (2.14) does not hold. In the specific example of Ref. [15], $(\delta_{12}^{LL})_{\rm eff} \sim \lambda^4$ and $B(\mu \to e\gamma)$ is close to the bound only if the sleptons are light, that is, $\tilde{m} \sim 100$ GeV.

3.5. Discrete symmetries

A discrete horizontal symmetry can lead to a large mixing simultaneously with large hierarchy by modifying the predictions of a continuous symmetry in one of the following three ways [15]:

1. Mass enhancement: $(M_\nu)_{33}$ is enhanced;
2. Mixing enhancement: $(M_{\ell^\pm})_{23}$ is enhanced;
3. See-saw suppression: singlet neutrino masses are enhanced, thus suppressing light neutrino masses.

In the models of Ref. [15], singlet neutrinos play no role. In the first two cases, the discrete symmetry induces $\sin\theta_{23}^\nu \ll 1$ and therefore the large AN mixing must arise in the charged lepton sector. Consequently, (2.12) necessarily holds. The third scenario is operative even in the case that $H(L_2) = H(L_3)$. The generic (though, in this case, not unavoidable) prediction is then again that (2.12) holds.

As concerns the 12 subspace, there is no generic prediction. The explicit models constructed in Ref. [15] include an MSW(SA) model where (2.14) holds and a VO model where (2.13) does not hold ($(\delta_{12}^{LL})_{\rm eff} \ll 1$).

3.6. Two breaking parameters

If a horizontal $U(1)$ symmetry is broken by two small parameters of equal magnitude and opposite charge [32], then a large mixing angle could arise also for $H(L_2) \neq H(L_3)$, leading to hierarchical neutrino masses. Models of this type that accommodate (1.1) and (1.2) have been constructed in Ref. [16]. In the models of [16], the large mixing in the 23 subspace is achieved by

$$|H(L_2) + H(\bar\ell_3)| = |H(L_3) + H(\bar\ell_3)|. \tag{3.3}$$

Then (3.1) holds with the resulting predictions for $B(\tau \to \mu\gamma)$.

As concerns the 12 subspace, there is again no generic prediction. In one model of Ref. [16], the VO solution is accommodated using the same mechanism to induce the large 12 mixing as the 23 mixing, that is, $|H(L_1) + H(\bar\ell_2)| = |H(L_2) + H(\bar\ell_2)|$. Consequently,

$$(M_\ell)_{12}/(M_\ell)_{22} \sim 1 \implies (\delta_{12}^{LL})_{\rm eff} \sim 1, \quad (\delta_{12}^{LR})_{\rm eff} \sim 10^{-3}. \tag{3.4}$$

In other words, the naive estimate of Eq. (2.13) holds and the model is excluded.

In another model of Ref. [16], the VO parameters are related to a pseudo-Dirac structure in the 12 subspace. The charged lepton mass matrix is near-diagonal in the 12 subspace and, consequently, $(\delta_{12}^{MN})_{\rm eff}$ is negligibly small.

To summarize this section: in all the models that have been proposed in the literature, the charged lepton sector has a large 23 mixing. Our naive estimate (2.12) holds. If $\tilde{m} \sim 100$ GeV, then $B(\tau \to \mu\gamma)$ is close to the upper bound.

On the other hand, there is a large variety of predictions concerning $B(\mu \to e\gamma)$. Some models are excluded because they predict this rate to be above the present bound by many orders of magnitude. Others are viable only if $\tilde{m} \gtrsim 500$ GeV and predict that the rate of radiative muon decay is close to the bound. Finally, there are models in which the charged lepton sector has a negligible 12 mixing, predicting $B(\mu \to e\gamma)$ well below the present bound.

4. Avoiding $(\delta_{23}^{LL})_{\rm eff} \sim 1$

In our survey of the literature in the previous section, we have only encountered models where the naive predictions (2.12) hold and, therefore, if slepton masses are of order 100 GeV, the rate of $\tau \to \mu\gamma$ is predicted to be close to the bound. This is not an accidental result:

(i) In many models, $H(L_2) = H(L_3)$. Then, large $(\delta_{23}^{LL})_{\rm eff}$ is unavoidable.

(ii) In many models, the hierarchy in the neutrino sector is closely related to a near-diagonal structure of the neutrino mass matrix in the 23 subspace. Then the charged lepton sector must account for the large 23 mixing and $(\delta_{23}^{LL})_{\rm eff} \sim 1$ is unavoidable.

The second argument holds, however, only in models where singlet neutrinos play no role. More precisely, it is valid in models where the AN parameters are determined by the horizontal charges of L_2, L_3, $\bar{\ell}_2$, $\bar{\ell}_3$ and the Higgs doublets only. (In Ref. [16], these models are called (2,0) models, where the first integer refers to the number of active neutrinos, and the second to the number of sterile.) In such models, the selection rules apply directly to the Majorana mass matrix of the active neutrinos, and it is easy to see that the neutrino mass matrix cannot give both large mixing and hierarchically separated masses: Large 23 mixing would require $(M_\nu)_{23} \sim (M_\nu)_{33}$. Since an Abelian symmetry cannot relate the coefficients of order one, this then implies that in the 23 subspace $\det M_\nu \sim [(M_\nu)_{33}]^2$, and the two mass eigenvalues are of the same order of magnitude.

As we will see below, however, if singlet neutrinos play a role in determining the light neutrino masses and mixings, it is possible to obtain both large mixing and mass hierarchies from the neutrino matrix alone. In this case, there need not be large mixings in the charged lepton sector. We will first argue that the addition of singlet neutrinos by itself cannot lead to a situation where $V^\ell_{23} \sim 1$ and $(\delta^{LL}_{23})_{\rm eff} \ll 1$; an additional special ingredient, such as holomorphic zeros, is required. Then we give three examples of such models, each related to a different framework presented in the previous section. These illustrative examples are (2,2) models. In the final subsection, we show that it is also possible to suppress $(\delta^{LL}_{23})_{\rm eff}$ in three generation models. We first give a (3,3) example. We then present a viable (3,0) model, that is, a model where singlet neutrinos play no role but the horizontal charge of L_1 affects the masses and/or mixing of ν_2 and ν_3.

4.1. Naive (3,3) models

We now argue that if we only modify our naive models, defined in section 2.3, by the addition of singlet neutrinos, then we cannot achieve $V^\ell_{23} \sim 1$ and $(\delta^{LL}_{23})_{\rm eff} \ll 1$. To be specific, we consider a (3,3) model with the following features:

a. The horizontal symmetry is a single $U(1)$;
b. The symmetry is broken by a single parameter $\lambda(-1)$;
c. There are three active and three singlet neutrinos.
d. There are no holomorphic zeros in either $M_\nu^{\rm Dir}$ or $M_{\nu_s}^{\rm Maj}$.

We denote the Majorana mass matrix for the *light* neutrinos by M_ν. It is given by

$$M_\nu = M_\nu^{\rm Dir} (M_{\nu_s}^{\rm Maj})^{-1} (M_\nu^{\rm Dir})^T. \tag{4.1}$$

In Eq. (2.8) we estimated M_ν assuming that singlet neutrinos play no role. Here we will prove that the same estimate applies also to M_ν of Eq. (4.1) if the conditions a-d hold.

If there are no holomorphic zeros in the neutrino mass matrices, then we have

$$(M_{\nu_s}^{\rm Maj})_{ij} \sim M \lambda^{H(N_i)+H(N_j)}, \tag{4.2}$$

and

$$(M_\nu^{\rm Dir})_{ij} \sim v_u \lambda^{H(L_i)+H(N_j)+H(\phi_u)}. \tag{4.3}$$

We recall that for any non-singular 3×3 matrix,

$$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \tag{4.4}$$

we have

$$A^{-1}=\frac{1}{\det A}\begin{pmatrix} a_{22}a_{33}-a_{23}a_{32} & a_{13}a_{32}-a_{12}a_{33} & a_{12}a_{23}-a_{13}a_{22} \\ a_{23}a_{31}-a_{21}a_{33} & a_{11}a_{33}-a_{13}a_{31} & a_{13}a_{21}-a_{11}a_{23} \\ a_{21}a_{32}-a_{22}a_{31} & a_{12}a_{31}-a_{11}a_{32} & a_{11}a_{22}-a_{12}a_{21} \end{pmatrix}. \tag{4.5}$$

Taking into account that $M_{\nu_s}^{\rm Maj}$ is symmetric, Eqs. (4.2) and (4.5) lead straightforwardly to [33]

$$[(M_{\nu_s}^{\rm Maj})^{-1}]_{ij}\sim[(M_{\nu_s}^{\rm Maj})_{ij}]^{-1}\sim\frac{1}{M}\lambda^{-H(N_i)-H(N_j)}. \tag{4.6}$$

From Eqs. (4.1), (4.3) and (4.6), we find:

$$(M_\nu)_{ij}\sim\frac{v_u^2}{M}\lambda^{H(L_i)+H(L_j)+2H(\phi_u)}. \tag{4.7}$$

We find then that indeed (2.8) holds for this case in spite of the presence of singlet neutrinos. As a result of (4.7), we have

$$\sin\theta_{23}^{\nu}\sim 1 \implies H(L_2)=H(L_3). \tag{4.8}$$

Consequently, $\sin\theta_{23}^{\ell}\sim 1$ is unavoidable, leading to $(\delta_{23}^{LL})_{\rm eff}\sim 1$. We conclude that, to suppress $(\delta_{23}^{LL})_{\rm eff}$, the naive models have to be modified beyond the addition of singlet neutrinos.

It is important to note that our proof here applies for any number of singlet neutrinos. This is straightforward to see, using the generalization of (4.5) to an $n_s\times n_s$ matrix.

4.2. Holomorphic zeros

We now consider again models with a horizontal $U(1)_1\times U(1)_2$ symmetry, where each of the $U(1)$ factors is broken by a single small parameter, $\epsilon_1(-1,0)\sim\lambda^m$ and $\epsilon_2(0,-1)\sim\lambda^n$. In such models, neutrino mass matrices with hierarchical masses (and weak mixing) are easily achieved if $H_1(L_2)\neq H_1(L_3)$ and $H_2(L_2)\neq H_2(L_3)$. If in addition L_2 and L_3 have equal effective horizontal charge,

$$H_{\rm eff}(L_2)=H_{\rm eff}(L_3),\quad (H_{\rm eff}=mH_1+nH_2), \tag{4.9}$$

large 23 mixing in the charged lepton matrix can also be arranged. In Ref. [15], this mechanism was employed in the framework of (2,0) models. Such models then satisfy the condition of large mixing and large hierarchy in the 23 neutrino sector, but, as discussed above, also predict $\tau \to \mu\gamma$ rates near the present bound.

We now consider $(2, n_s \geq 2)$ models, that is, models where the charges of at least two singlet neutrinos do affect the AN parameters. In this case, the light neutrino mass matrix has the form of Eq. (4.1). We would like to obtain both large mixing and large hierarchy from the neutrino matrix alone. The exact conditions for this to hold are complicated, but it is easy to show that a sufficient condition is

$$\begin{aligned} (M_\nu^{\rm Dir})_{23} \sim (M_\nu^{\rm Dir})_{33} &\gg \text{all other entries of } M_\nu^{\rm Dir} \\ (M_{\nu_s}^{\rm Maj})^{-1}_{33} &\gg \text{all other entries of } (M_{\nu_s}^{\rm Maj})^{-1}. \end{aligned} \tag{4.10}$$

To reduce mixing in the charged lepton matrix, we may use holomorphy to produce

$$(M_{\ell^\pm})_{23} = 0, \quad (M_{\ell^\pm})_{33} \neq 0. \tag{4.11}$$

As an example of how this mechanism works, we now present an explicit (2,2) model. We take $m = n = 1$, that is $\epsilon_1 \sim \epsilon_2 \sim \lambda$, and assign the following set of charges for the lepton fields:

$$L_2(-1,0), \quad L_3(-2,1), \quad N_2(0,0), \quad N_3(2,0), \quad \bar\ell_2(1,5), \quad \bar\ell_3(5,-1). \tag{4.12}$$

The 2×2 mass matrices in the 23 subspace have the following forms:

$$M_\nu^{\rm Dir} \sim v_u \begin{pmatrix} 0 & \lambda \\ 0 & \lambda \end{pmatrix}, \quad M_{\nu_s}^{\rm Maj} \sim M \begin{pmatrix} 1 & \lambda^2 \\ \lambda^2 & \lambda^4 \end{pmatrix}, \quad M_{\ell^\pm} \sim v_d \begin{pmatrix} \lambda^5 & 0 \\ 0 & \lambda^3 \end{pmatrix}. \tag{4.13}$$

These matrices satisfy the conditions of Eqs. (4.10) and (4.11). For the mass ratios and mixing, we get the following estimates:

$$m(\nu_2)/m(\nu_3) = 0, \quad m(\ell_2^\pm)/m(\ell_3^\pm) \sim \lambda^2, \quad V_{23}^\ell \sim 1. \tag{4.14}$$

(The vanishing neutrino mass will be lifted when the first generation is incorporated.) Note that the charged lepton mass matrix is diagonal in the 23 subspace, and so the neutrino mixing is generated completely by the neutrino mass matrix. Equation (4.13) leads to

$$(\delta_{23}^{LL})_{\rm eff} \sim \lambda^2, \tag{4.15}$$

and $B(\tau \to \mu\gamma)$ well below the bound.

4.3. Discrete symmetries

In models with a discrete $Z_p \times U(1)$ symmetry, it is possible to enhance a light neutrino mass eigenvalue if some of the entries in the neutrino mass matrix are larger than their would-be value if the symmetry were continuous. In Ref. [15], this mechanism was employed in the framework of (2,0) models to build viable models of neutrino parameters, where the large mixing must come from the charged lepton sector.

In $(2, n_s \geq 2)$ models one can also use the mechanism of discrete symmetries to induce $V_{23}^{\ell} \sim 1$ from the neutrino mass matrix and not from the charged lepton mass matrix. In particular, we can arrange for the conditions of Eq. (4.10) to hold again for the neutrino mass matrix, while arranging for the discrete symmetry to produce

$$(M_{\ell^\pm})_{23} \ll (M_{\ell^\pm})_{33}. \tag{4.16}$$

As an example, we now present an explicit (2,2) model. We take $p = 5$, that is a $Z_5 \times U(1)$ symmetry, and $m = n = 1$. We assign the following set of charges for the lepton fields:

$$L_2(3,1), \quad L_3(4,0), \quad N_2(0,0), \quad N_3(2,0), \quad \bar{\ell}_2(2,4), \quad \bar{\ell}_3(1,3). \tag{4.17}$$

The 2×2 mass matrices in the 23 subspace have the following forms:

$$M_\nu^{\rm Dir} \sim v_u \begin{pmatrix} \lambda^4 & \lambda \\ \lambda^4 & \lambda \end{pmatrix}, \quad M_{\nu_s}^{\rm Maj} \sim M \begin{pmatrix} 1 & \lambda^2 \\ \lambda^2 & \lambda^4 \end{pmatrix}, \quad M_{\ell^\pm} \sim v_d \begin{pmatrix} \lambda^5 & \lambda^8 \\ \lambda^5 & \lambda^3 \end{pmatrix}, \tag{4.18}$$

again satisfying Eq. (4.10). For the mass ratios and mixing, we get the following estimates:

$$m(\nu_2)/m(\nu_3) \sim \lambda^{10}, \quad m(\ell_2^\pm)/m(\ell_3^\pm) \sim \lambda^2, \quad V_{23}^{\ell} \sim 1, \tag{4.19}$$

where the source of the large neutrino mixing is the neutrino mass matrix. Note that (4.16) is satisfied, and Eq. (4.18) leads to

$$(\delta_{23}^{LL})_{\rm eff} \sim \lambda^2 \tag{4.20}$$

and $B(\tau \to \mu\gamma)$ well below the bound.

4.4. Two breaking parameters

Finally, we consider models in which a horizontal $U(1)$ symmetry is broken by two parameters of opposite charge and equal magnitude ($\lambda(-1)$ and $\bar{\lambda}(+1)$). In Ref. [16], viable models that employ this mechanism with the condition (3.3) were constructed with large 23 mixing required to diagonalize the charged lepton sector.

In $(2, n_s \geq 2)$ models we can use the mechanism of two breaking parameters to satisfy Eq. (4.10) by imposing

$$|H(L_2) + H(N_3)| = |H(L_3) + H(N_3)|. \tag{4.21}$$

To suppress mixing in the charged lepton sector, it is sufficient to note that in the generic case $H(N_3) \neq H(\bar{\ell}_3)$, and so (3.3) does not hold and consequently also (4.16) is satisfied.

As an example of how this mechanism works, we present an explicit (2,2) model. We assign the following set of charges for the lepton fields:

$$L_2(-1), \quad L_3(-3), \quad N_2(0), \quad N_3(2), \quad \bar{\ell}_2(-4), \quad \bar{\ell}_3(6). \tag{4.22}$$

The 2×2 mass matrices in the 23 subspace have the following forms:

$$M_\nu^{\rm Dir} \sim v_u \begin{pmatrix} \lambda & \lambda \\ \lambda^3 & \lambda \end{pmatrix}, \quad M_{\nu_s}^{\rm Maj} \sim M \begin{pmatrix} 1 & \lambda^2 \\ \lambda^2 & \lambda^4 \end{pmatrix}, \quad M_{\ell^\pm} \sim v_d \begin{pmatrix} \lambda^5 & \lambda^5 \\ \lambda^7 & \lambda^3 \end{pmatrix}. \tag{4.23}$$

The mass ratios and mixing are

$$m(\nu_2)/m(\nu_3) \sim \lambda^4, \quad m(\ell_2^\pm)/m(\ell_3^\pm) \sim \lambda^2, \quad V_{23}^\ell \sim 1. \tag{4.24}$$

where again, the $\mathcal{O}(1)$ neutrino mixing is from the neutrino mass matrix. Equation (4.23) leads to

$$(\delta_{23}^{LL})_{\rm eff} \sim \lambda^2, \tag{4.25}$$

and $B(\tau \to \mu\gamma)$ well below the bound.

4.5. Three Generation Models

In the previous sections, we have constructed a variety of (2,2) examples. It is possible to extend these to (3,3) models that fit both the AN and SN parameters and where the radiative charged lepton decays are suppressed. For example, in the two breaking parameter

framework, if we assign charges

$$L_1(+1),\ L_2(-3),\ L_3(-1),\ N_1(0),\ N_2(0),\ N_3(+2),\ \bar{\ell}_1(-6),\ \bar{\ell}_2(5),\ \bar{\ell}_3(1), \tag{4.26}$$

we find $\Delta m^2_{12}/\Delta m^2_{23} \sim \lambda^8$, $V^\ell_{12} \sim 1$ and $V^\ell_{23} \sim 1$, as appropriate for the AN and for the VO solution of the SN. We also have $V_{e3} \ll 1$, consistent with CHOOZ and AN, and contributions to $\tau \to \mu\gamma$ and $\mu \to e\gamma$ well below bounds. We did not prove, however, that all the mechanisms discussed in this section can be extended to the three generation case, nor have we shown that all three of the SN solutions can be accommodated in such models.

Finally, we note that it is possible to achieve the AN and SN parameters (1.1) and (1.2) together with a suppressed $(\delta^{LL}_{23})_{\rm eff}$ in models without singlet neutrinos but with the horizontal charges of all three active neutrinos playing a role in achieving the AN parameters.

We now give an explicit example of such a (3,0) model. The horizontal symmetry is $U(1)$, with two small breaking parameters, similar to the previous subsection. We assign the following set of charges for the lepton fields:

$$L_1(-4),\quad L_2(6),\quad L_3(2),\quad \bar{\ell}_1(9),\quad \bar{\ell}_2(-4),\quad \bar{\ell}_3(-2). \tag{4.27}$$

The 3×3 mass matrices have the following forms:

$$M^{\rm Maj}_{\nu_a} \sim \frac{v_u^2}{M} \begin{pmatrix} \lambda^8 & \lambda^2 & \lambda^2 \\ \lambda^2 & \lambda^{12} & \lambda^8 \\ \lambda^2 & \lambda^8 & \lambda^4 \end{pmatrix}, \quad M_{\ell^\pm} \sim v_d \begin{pmatrix} \lambda^5 & \lambda^8 & \lambda^6 \\ \lambda^{15} & \lambda^2 & \lambda^4 \\ \lambda^{11} & \lambda^2 & 1 \end{pmatrix}. \tag{4.28}$$

For the mass ratios and mixing, we get the following estimates:

$$\Delta m^2_{12}/\Delta m^2_{23} \sim \lambda^4,\quad m(\ell^\pm_2)/m(\ell^\pm_3) \sim \lambda^2,\quad V^\ell_{23} \sim 1. \tag{4.29}$$

However, the mixing in the charged lepton matrix is suppressed, with

$$(\delta^{LL}_{23})_{\rm eff} \sim \lambda^4,\ (\delta^{RR}_{23})_{\rm eff} \sim \lambda^2, \tag{4.30}$$

and $B(\tau \to \mu\gamma)$ well below the bound.

5. Conclusions

We have studied the predictions of supersymmetric Abelian flavor models for lepton flavor violating decays in view of measurements of neutrino mass and mixing parameters. We have found no model-independent predictions, and so are unable to conclude that charged lepton flavor violating processes provide unambiguous tests of this framework. However, we can make the following interesting observations:

1. For models without singlet neutrinos, a generic prediction is that $B(\tau \to \mu\gamma)$ is close to its current bound for slepton masses of order 100 GeV and may be within reach of future experiments.
2. Conversely, if slepton masses are found to be not too heavy and the upper bound on $B(\tau \to \mu\gamma)$ becomes stronger, then typically rather complicated models, for example, those constructed above involving singlet neutrinos in an essential way, are required and the Abelian symmetry framework loses predictive power.
3. Many models where the solar neutrino problem is solved by large mixing (either vacuum oscillations or large angle matter enhanced oscillations) are excluded or strongly disfavored by current bounds on $B(\mu \to e\gamma)$ and $\mu - e$ conversion.
4. In models where the solar neutrino problem is solved by small angle matter enhanced oscillations, current bounds on $B(\mu \to e\gamma)$ and $\mu - e$ conversion often already require slepton masses to be above $\mathcal{O}(500\text{ GeV})$. In such models, large signals are predicted in future experiments sensitive to $\mu \to e\gamma$ and $\mu - e$ conversion. Given the projected improvements of three to four orders of magnitude, such experiments are extremely interesting and promising.
5. Conversely, if no signal appears in future experiments probing $\mu \to e\gamma$ and $\mu - e$ conversion, many models will be excluded, and the framework of Abelian horizontal symmetries is again typically required to become rather baroque and of limited predictive power.
6. If experiments favor the vacuum oscillations solution with near-maximal mixing, then Abelian flavor symmetries that lead to a pseudo-Dirac structure in the 12 subspace of the neutrino mass matrix provide an attractive explanation.

Acknowledgments

J.L.F. is supported by the Department of Energy under contract No. DE–FG02–90ER40542 and through the generosity of Frank and Peggy Taplin. Y.N. is supported by the Department of Energy under contract No. DE–FG02–90ER40542, by the Ambrose Monell Foundation, by the United States – Israel Binational Science Foundation (BSF), by the Israel Science Foundation founded by the Israel Academy of Sciences and Humanities, and by the Minerva Foundation (Munich). Y.S. is supported in part by the Koret Foundation.

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